

GCE

Further Mathematics A

Y535/01: Additional Pure Mathematics

Advanced Subsidiary GCE

2020 Mark Scheme (DRAFT)

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Question			Answer	Marks	AO	Guidance
1	(a)		$30 \pmod{31}$ or $-1 \pmod{31}$	B1 [1]	1.1	BC No other answer to be accepted Note: $13 \times 19 = 247 = 7 \times 31 + 30 \equiv 30 \pmod{31}$
	(b)		$13x \equiv 9 \equiv 40 \equiv 71 \equiv \dots \equiv 195$ so $x \equiv 15 \pmod{31}$ OR $x = 31n + 15$	M1 A1 A1	1.1 1.1 2.2a	Repeatedly adding 31s arriving at a multiple of 13 $n \in \mathbb{Z}$ need not be stated
			Alternative method $13 \times 19 \equiv -1 \Rightarrow 13 \times (19 \times 13 \times 19) \equiv 1$ so $19 \times 13 \times 19 \equiv 12$ is the reciprocal of 13 (mod 31) Then $12 \times 13x \equiv 12 \times 9$ $\Rightarrow x \equiv 15 \pmod{31}$	M1 M1 A1		Method for finding reciprocal (inverse) of 13 (mod 31) using (a) Multiplication by the reciprocal correct answer
				[3]		

Question			Answer	Marks	AO	Guidance
2	(a)		$xyh = 1000 \Rightarrow h = \frac{1000}{xy}$ $A = xy + 2xh + 2yh$ $= xy + 2000\left(\frac{1}{x} + \frac{1}{y}\right)$	B1 B1 M1 A1 [4]	3.1b 1.1 2.1 1.1	soi Substitution of h expression from (a) (i) AG shown with supporting working
	(b)	(i)	$\frac{\partial A}{\partial x} = y + 2000\left(\frac{-1}{x^2}\right)$ and $\frac{\partial A}{\partial y} = x + 2000\left(\frac{-1}{y^2}\right)$ Both p.d.s set to zero and solving $x = y = 10 \times 2^{\frac{1}{3}}$	M1 A1 B1 M1 A1 [5]	1.1 1.1 1.1 2.1 1.1	Partially differentiating A w.r.t. x or y ; either correct 2 nd correct: FT 1 st , with $x \leftrightarrow y$ $x^2y = xy^2 = 2000$ Both correct
		(ii)	Substg. x, y back into formula for A ; $300 \times 2^{\frac{2}{3}}$	M1 A1 [2]	1.1 1.1	Any exact equivalent e.g. $150 \times 2^{\frac{5}{3}}$, $75 \times 2^{\frac{8}{3}}$ or awrt 476 BC
3	(a)		13 divides each pair of digits of N (26, 13, 26, 52)	B1 [1]	2.4	Or applying a standard divisibility test
	(b)		$4 \mid 52$ (the final two digits of N) $\Rightarrow 4 \mid N$ $9 \mid$ digit-sum of N ($= 27$) $\Rightarrow 9 \mid N$ Since $\text{hcf}(4, 9) = 1$, $4 \times 9 = 36 \mid N$	B1 B1 B1 [3]	1.1 1.1 2.4	Applying these two divisibility tests Must explain that 4, 9 are co-prime as well as state the conclusion
	(c)		By <i>Euclid's Lemma</i> , $13 \mid 36 \times 725907$ and $\text{hcf}(13, 36) = 1$ $\Rightarrow 13 \mid 725907$	M1 A1 [2]	2.4 2.2a	M for stating “Euclid’s Lemma” (or full description of its result) Clear outline of necessary conditions

Question			Answer							Marks	AO	Guidance																																											
4	(a)		<table><tr><td>$\times_{14}$</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr><tr><td>2</td><td>4</td><td>8</td><td>12</td><td>2</td><td>6</td><td>10</td></tr><tr><td>4</td><td>8</td><td>2</td><td>10</td><td>4</td><td>12</td><td>6</td></tr><tr><td>6</td><td>12</td><td>10</td><td>8</td><td>6</td><td>4</td><td>2</td></tr><tr><td>8</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr><tr><td>10</td><td>6</td><td>12</td><td>4</td><td>10</td><td>2</td><td>8</td></tr><tr><td>12</td><td>10</td><td>6</td><td>2</td><td>12</td><td>8</td><td>4</td></tr></table>	$\times_{14}$	2	4	6	8	10	12	2	4	8	12	2	6	10	4	8	2	10	4	12	6	6	12	10	8	6	4	2	8	2	4	6	8	10	12	10	6	12	4	10	2	8	12	10	6	2	12	8	4	B1	1.1	For any two lines (Rs or Cs) correct
		$\times_{14}$	2	4	6	8	10	12																																															
		2	4	8	12	2	6	10																																															
		4	8	2	10	4	12	6																																															
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		10	6	12	4	10	2	8																																															
	12	10	6	2	12	8	4																																																
		B1	1.1	For at least two Rs and two Cs correct																																																			
		B1	1.1	For LSP applying to complete table																																																			
	B1	1.1	For symmetry about main diagonal																																																				
			(Must be fully correct for all 4 marks)																																																				
			[4]																																																				
(b)		Closed since no other elements appear in the table Identity is 8 Inverses: 6 is self-inverse $2^{-1} = 4$ and $4^{-1} = 2$; $10^{-1} = 12$ and $12^{-1} = 10$ (Hence a group)	B1 B1 B1 B1	2.4 2.2a 1.2 2.5	Don't accept "closed, from table" only Any clear indication of inverses (not just statement they exist) That is, (2, 4) and (10, 12) are inverse-pairs Associativity and conclusion not required																																																		
			[4]																																																				
(c)	(i)	{8, 6} {8, 2, 4}	B1 B1 [2]	2.2a 1.1	One correct; both (and no extras). Ignore {8} and G																																																		
	(ii)	10, 12	B1 B1 [2]	1.1 1.1	One correct; both (and no extras)																																																		
5	(a)		Complementary Solution is $V_n = A \times 2^n$ For Particular Solution, try $V_n = an + b$ Then $V_{n+1} = 2V_n + n \Rightarrow an + (a + b) = 2an + 2b + n$ Comparing coefficients: $a = 2a + 1$ and $a + b = 2b$ $\Rightarrow a = b = -1$ General Solution is thus $V_n = A \times 2^n - n - 1$	B1 M1 A1 M1 A1 B1	1.2 1.1a 1.1 1.1 1.1 1.1	Allow $V_n = an$ for method mark Substitution and comparing of coefficients FT GS = CS + PS provided CS has one arbitrary constant and PS has none (and is a polynomial)																																																	
					[6]																																																		

Question			Answer	Marks	AO	Guidance
5	(b)		$V_1 = 8 \Rightarrow A = 5$ so $V_n = 5 \times 2^n - n - 1$	M1	3.1a	soi (or BC)
			So $V_{20} = 5\,242\,859$	A1 [2]	1.1	accept exact value only.
6	(a)		$\mathbf{a} \times \mathbf{b} = -14\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$	B1	1.1	A correct vector product (possibly BC)
			Use of formula Area $\Delta = \frac{1}{2} \mathbf{a} \times \mathbf{b} $	M1	1.1	Including an attempt at a vector product
			Area $\Delta OAB = 5\sqrt{3}$	A1	1.1	Accept alternative exact equivalents (e.g. $\sqrt{75}$)
				[3]		
	(b)		$(\mathbf{r} - \mathbf{a}) \times (\mathbf{b} - \mathbf{a}) = \mathbf{0}$ is the line through A and B	M1	2.2a	From this point on, work may appear with numerical equivalent set-out
			so $\mathbf{c} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ or $\mathbf{c} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$	A1	3.1a	
			Area $\Delta OAC = \frac{1}{2} \mathbf{a} \times \mathbf{c} = \frac{1}{2} (1 - \lambda)\mathbf{a} \times \mathbf{a} + \lambda \mathbf{a} \times \mathbf{b} $	M1	2.1	
			$= \frac{1}{2} \mathbf{0} + \lambda \mathbf{a} \times \mathbf{b} $	M1	3.1a	
			Area $\Delta OAC = \frac{1}{2}$ Area $\Delta OAB \Rightarrow \lambda = \pm \frac{1}{2}$	A1	1.1	
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1	2.1	
			Alternative method			
			C is on the line AB	B1		
			Common “base” OA means that C is either the internal or the external bisector of AB	M1 A1		
			i.e. $\mathbf{c} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ or $\frac{1}{2}(3\mathbf{a} - \mathbf{b})$	M1 A1		
			giving $\mathbf{c} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	A1		
				[6]		

Question			Answer	Marks	AO	Guidance
7	(a)	(i)	E.g. $T_0 = 100\,000$ is the initial population as given $T_{k+1} = (1 - r) T_k$ because a death-rate of r means that $1 - r$ of the population is left after each week. $0 \leq k \leq 12$ because the model given is only valid for twelve weeks.	B1 B1 B1 [3]	1.1 3.3 2.1	
		(ii)	$T_{12} = a^{12} T_0$ $1 - r = \sqrt[12]{0.00355} = 0.62496 \dots \Rightarrow r = 0.375$ to 3s.f.	M1 A1 [2]	3.1b 1.1	$a = r$ or $1 - r$ AG
	(b)	(i)	After 16 weeks, the number of frogs is $0.62496 \dots^{16} \times 100\,000 = 54.154 \dots$ So $54.154 \dots \times p \geq 30$ $\Rightarrow p \geq \frac{30}{54.154 \dots} = 0.5539 \dots = 0.554$ to 3 sf	B1 M1 A1 [3]	3.5c 3.1b 1.1	Allow use of ' T_{16} '. Or, starting again $0.62496 \dots^4 \times 355$ For 'their population' $\times p \geq 30$
		(ii)	E.g. The same weekly death-rate factor continues unchanged. The females will all lay eggs. Tadpoles instantly change to frogs and lay eggs at exactly the same time.	B1 [1]	3.3	
	(c)		E.g. 30 surviving females would produce 75000 eggs, so the population is smaller than it was to start with, so each 'round' will result in smaller and smaller populations.	B1 [1]	3.5a	No greater detail of analysis is required beyond "they would appear to be dying out so the figure of 30 in the model is not a good one"

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